Simple Harmonic Motion & Springs

Week 13, Lesson 1

- **Period & frequency**
- **Graph of Vibratory Motion**
- **Displacement, Restoring Force, Hooke's Law**
- **Period & Acceleration in SHM**
- **The Simple Pendulum**

References/Reading Preparation:

Schaum's Outline Ch. 11

Principles of Physics by Beuche – Ch.14

Simple Harmonic Motion

A vibrating system is one which undergoes the same motion over and over again.

Examples are: a pendulum, a weight on a spring, among others.

The time it takes for one complete cycle (the combined back and forth motion, or up and down motion)
is called the **period**, T.

It is the total time for the combined back and forth motion.

The unit is the second, s.

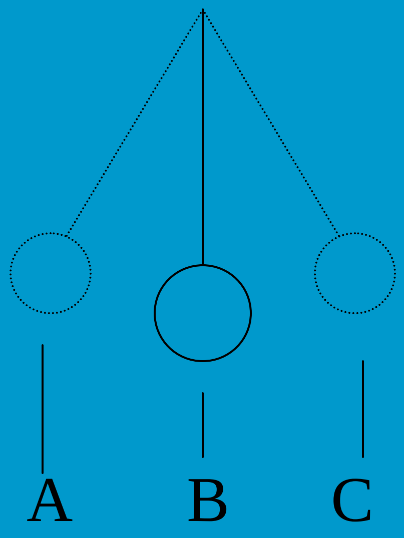
The number of cycles completed in a unit time is called the frequency f.

$$f = 1/T \qquad \text{(cycles/s)}$$

One cycles/s is equivalent to 1 hertz (Hz).

The amplitude is the maximum displacement of the object from the position the object maintains when it is not vibrating.

For a **pendulum**, the **amplitude** is the maximum ‘swing’ of the object. In the diagram below, it is the distance AB or BC.



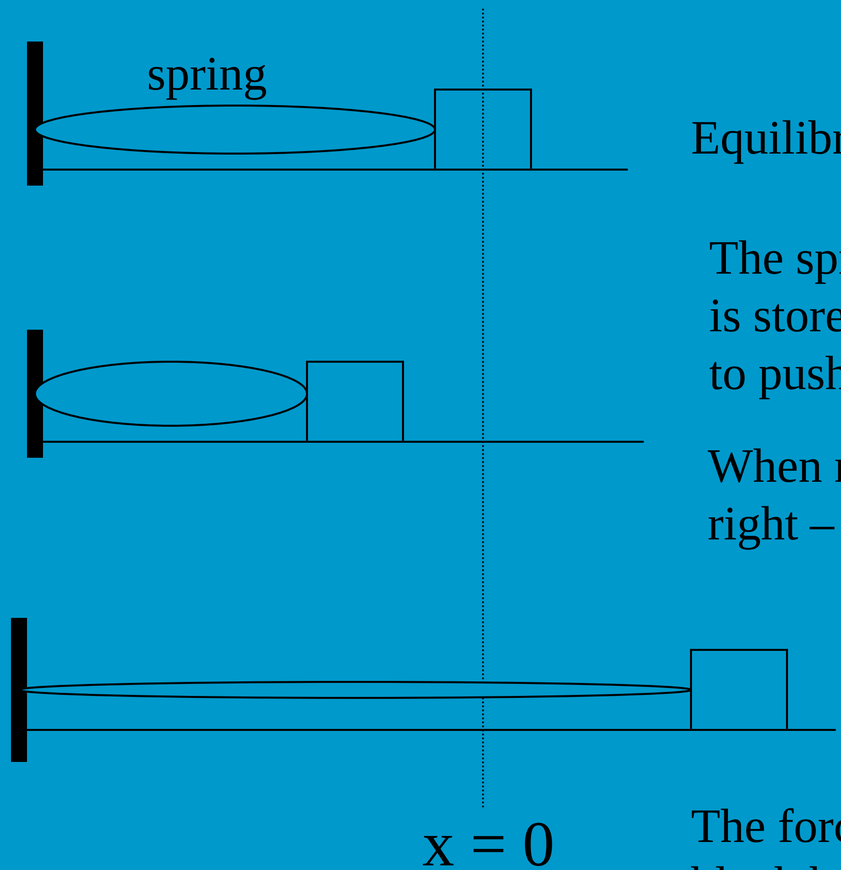
Note the important feature of the interchange of energy between potential and kinetic energy.

At A or C, the ball is at rest – therefore there is no KE, but max. GPE.

As it swings to B, it loses GPE and gains an equal amount of KE.

The same goes for a spring system with a mass at the end.

Consider the set-up shown with the mass sliding on a frictionless surface.



Equilibrium

The spring is compressed – therefore there is stored PE. The force in the spring wants to push the block back to x = 0.

When released, the block accelerates to the right – at x = 0, PE = 0, and KE is max.

The spring is stretched – therefore there is stored PE, and KE = 0

The force in the spring wants to pull the block back to equilibrium.

When the block is released, it will continue to oscillate back and forth if the surface is frictionless.

This is because the force due to the spring wants to push or pull the block back to the equilibrium position.

This force is called the **restoring force.**

A **restoring force** is necessary if vibration is to occur.
It is a force that is always directed so as to push or pull the system back to its equilibrium (normal resting) position.

The magnitude of the restoring force depends on the 'stiffness' of the spring – which is called the **spring constant** k.

Experiments have shown that the force in a spring is directly proportional to the displacement. They are related by the formula $F = -kx$, where F is the restoring force exerted by the spring and is in the opposite direction to the distortion.

This relationship is known as Hooke's Law.

Hooke's Law is obeyed by a system if the magnitude of The *restoring force* is proportional to the magnitude of the *displacement*.

The potential energy stored in the spring, distorted a distance x, is $\frac{1}{2} kx^2$, where k is the spring constant.

If the amplitude is x_o, then the energy of the vibrating system **at all times** is $\frac{1}{2} kx_o^2$.

From the law of conservation of energy,

KE + PE = constant

or

$$\boxed{\frac{1}{2} mv^2 + \frac{1}{2} kx^2 = \frac{1}{2} kx_o^2}$$

(where x_o is the amplitude)

From this we can determine the velocity of the vibrating object at any point during its displacement.

$$|v| = \sqrt{(x_o^2 + x^2)k/m}$$

Simple Harmonic Motion (SHM) is the vibratory motion that a system that obeys Hooke's Law undergoes.

Said another way:

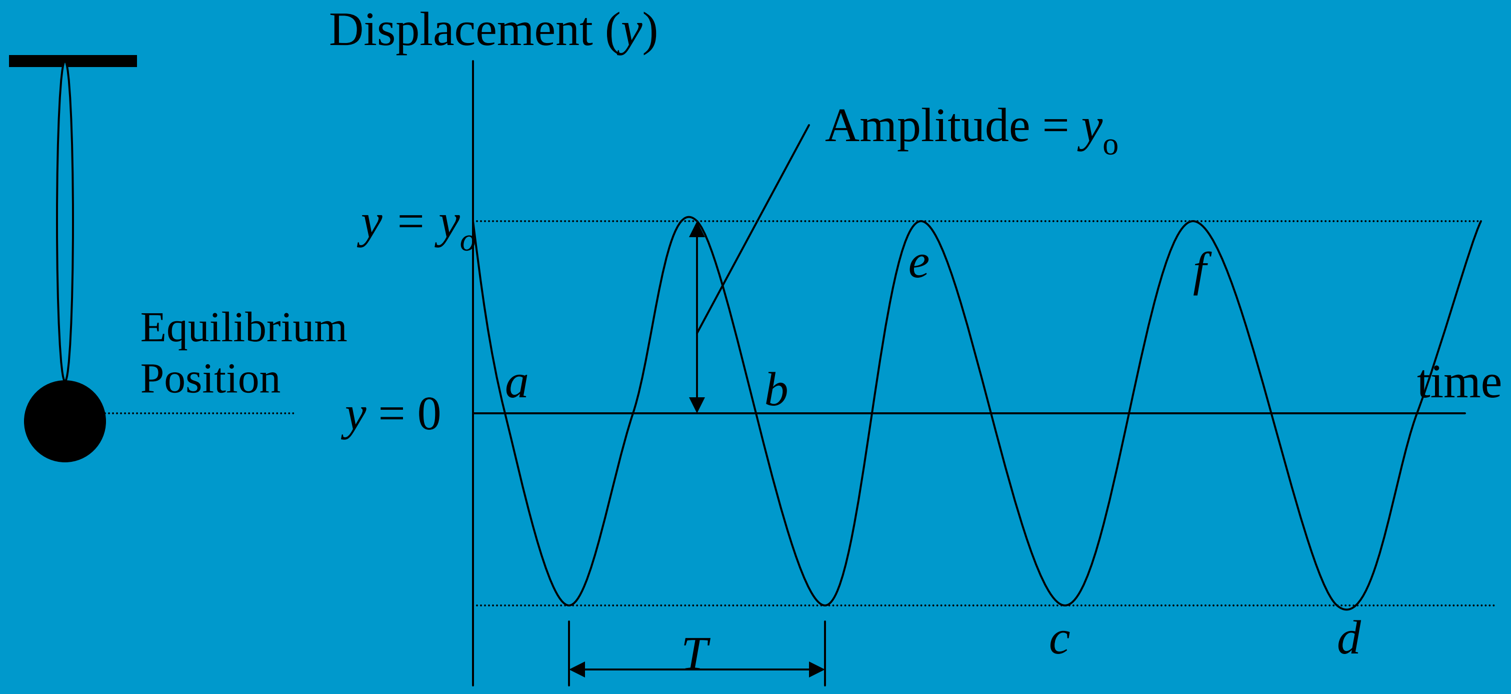
A vibratory motion that obeys Hooke's Law is said to be in Simple Harmonic Motion.

This motion is illustrated on the next slide. Because of the resemblance of its graph to a sine or cosine curve, SHM is frequently called *sinusoidal motion*.

The Graph of Vibratory Motion

The graph of vibratory motion is shown. The motion shown is the up-and-down motion of a mass at the end of a spring.

One complete cycle is from a to b, or from c to d, or from e to f.



The time for one cycle is T, the period.

Acceleration in SHM

Acceleration in SHM is given by Hooke's Law, $F = -kx$, and $F = ma$. Equating these two expressions for F gives

$$\boxed{a = -\frac{k}{m}x}$$

The minus sign indicates that the direction of **a** (and **F)** is always opposite to the direction of the displacement ***x***.

Period in SHM

The period *T* of a SHM is defined as:

$$\boxed{T = 2\pi \sqrt{m/k}}$$

Acceleration in terms of *T*

$$\boxed{a = -\frac{4\pi^2}{T^2}\, x}$$

The Simple Pendulum

The simple pendulum very nearly undergoes SHM if its angle of swing is not too large.

The period of vibration for a pendulum of length L at a location where the gravitational acceleration is g is given by:

$$\boxed{T = 2\pi \sqrt{L/g}}$$

Worked examples

A 50 g mass vibrates in SHM at the end of a spring. The amplitude of the motion is 12 cm, and the period is 1.70 s.

Find:

a) The frequency, *f*. (ans. 0.588 Hz)

b) The spring constant. (ans. 0.68 N/m)

c) The maximum speed of the mass. (ans. 0.44 m/s)

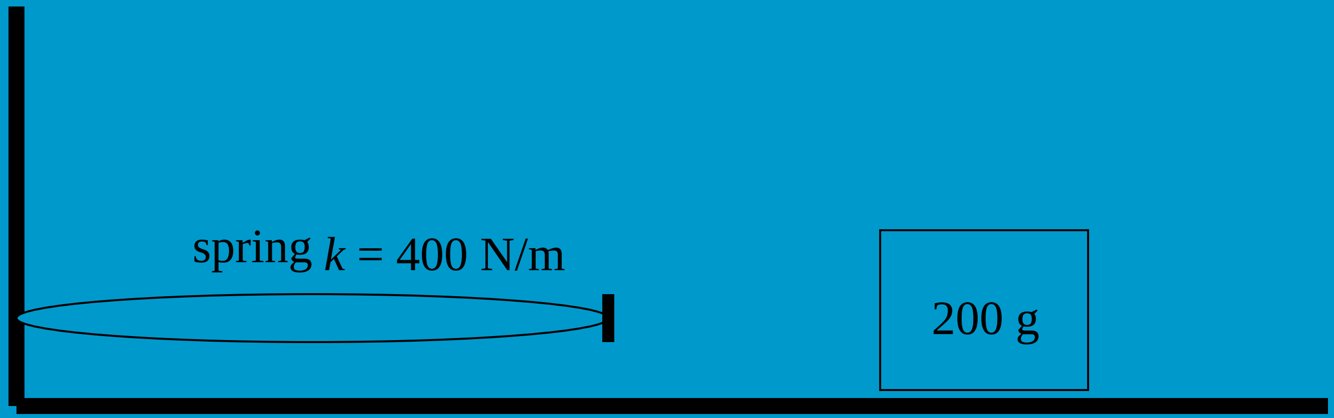
d) The maximum acceleration of the mass. (ans. 1.63 m/s^2)

e) The speed when the displacement is 6 cm. (ans. 0.38 m/s)

f) The acceleration when x = 6 cm. (ans. -0.82 m/s^2)

Worked example

The 200 g mass shown is pushed to the left against the spring and compresses the spring 15 cm from its equilibrium position. The system is then released, and the mass shoots to the right. If friction can be ignored, how fast will the mass be moving as it shoots away? Assume the mass of the spring to be very small.



(ans. 6.7 m/s)